

Natural Convection Flow Over A Parabolic Cylinder	العنوان:
Bany Youness, Asad Aldeen M. H.	المؤلف الرئيسي:
Haddad, Osamah Menwer(super)	مؤلفين آخرين:
2000	التاريخ الميلادي:
عمان، الأردن	موقع:
1 - 69	الصفحات:
568665	رقم MD:
رسائل جامعية	نوع المحتوى:
Arabic	اللغة:
رسالة ماجستير	الدرجة العلمية:
جامعة العلوم والتكنولوجيا الاردنية	الجامعة:
كلية الدراسات العليا	الكلية:
الاردن	الدولة:
Dissertations	قواعد المعلومات:
اسطوانات الحرارة، الفيزياء، الهندسة الميكانيكية	مواضيع:
<a href="https://search.mandumah.com/Record/568665">https://search.mandumah.com/Record/568665</a>	رابط:

# Natural Convection Flow Over a Parabolic Cylinder

By: Asad Al-deen M. H. Bany-Youness

Advisor : Dr. Osamah Haddad

## ABSTRACT

Steady, two-dimensional, symmetric and incompressible Natural convection flow over a parabolic cylinder was investigated numerically. The full Navier-Stokes and the energy equations were considered. These equations has been solved using a commercial code called 'COSMOS' which is based on finite element. Solutions for the temperature and velocity distributions are obtained for different values of the flow parameters. In addition, the local and average Nusselt number distributions are obtained and presented. For all cases, the surface was considered to be isothermal, and the following parameters are studied: nose radius of curvature of the parabolic body, Grashof number, and Prandtl number.

It was found that as the nose radius of curvature of the parabolic body is increased, the flow velocity will enhanced, but the local and average Nusselt number is decreased. Also, the increase in Grashof or Prandtl numbers will increase the local and average Nusselt number.

# انتقال الحرارة بالحمل الطبيعي من اسطوانة ذات مقطع قطع مكافئ.

إعداد : أسد الدين محمد حسين بني يونس .

إشراف : د. أسامة حداد.

## ملخص

لقد تم دراسة الجريان الطبيعي ذو البعدين و الثابت مع الزمن لموائع غير قابلة للانضغاط فوق أجسام ذات مقطع قطع مكافئ بواسطة استخدام معادلات الحركة بالإضافة إلى معادلة حفظ الطاقة. لقد حولت هذه المعادلات من معادلات مشتقة جزئية إلى مجموعة معادلات جبرية. و تم حل هذه المعادلات باستخدام برنامج يسمى كوزوموز. مما أمكن من الحصول على نتائج لتوزيع درجة الحرارة وسرعة السائل. وتم حساب رقم نسلت الموضعي، ومتوسط رقم نسلت.

لقد أعتبر توزيع الحرارة منتظماً على سطح جسم القطع المكافئ، المتغيرات التي تمت دراستها هي : نصف قطر الانحناء لمقدمة القطع المكافئ، رقم جرا شوف، رقم براندتل.

أظهرت الدراسة التأثير الواضح لقيم نصف قطر الانحناء لمقدمة القطع المكافئ، كذلك تأثير رقمي جرا شوف وبراندتل على الجريان في حالة الجريان الطبيعي، وفي النهاية تأثير المتغيرات السالفة الذكر على رقم نسلت الموضعي و المتوسط .

Natural Convection Flow Over A Parabolic Cylinder	العنوان:
Bany Youness, Asad Aldeen M. H.	المؤلف الرئيسي:
Haddad, Osamah Menwer(super)	مؤلفين آخرين:
2000	التاريخ الميلادي:
عمان، الأردن	موقع:
1 - 69	الصفحات:
568665	رقم MD:
رسائل جامعية	نوع المحتوى:
Arabic	اللغة:
رسالة ماجستير	الدرجة العلمية:
جامعة العلوم والتكنولوجيا الاردنية	الجامعة:
كلية الدراسات العليا	الكلية:
الاردن	الدولة:
Dissertations	قواعد المعلومات:
اسطوانات الحرارة، الفيزياء، الهندسة الميكانيكية	مواضيع:
<a href="https://search.mandumah.com/Record/568665">https://search.mandumah.com/Record/568665</a>	رابط:

## NOMENCLATURE

- $A_{ii}$  The algebraic coefficients.
- $C_f$  Local Skin friction,  $C_f = \frac{\tau_w}{\rho U_\infty^2}$ .
- $g$  Gravitational acceleration, [m/s<sup>2</sup>].
- $Gr_x$  Local Grashof number,  $Gr_x = \frac{g\beta \Delta T x^3}{\nu^2}$ .
- $h$  Local heat transfer coefficient, [w/ m k].
- $L$  Characteristic length of the parabola.
- $Nu$  Local Nusselt number at the surface of the parabola.
- $\overline{Nu}$  Average Nusselt number,  $\overline{Nu} = \frac{1}{L} \int_0^L Nu_x dx$ .
- $Pr$  Prandtl number,  $Pr = \frac{\nu}{\alpha}$ .
- $R$  Nose radius of curvature.
- $Re$  Reynolds number based on nose radius of curvature,  $Re = \frac{U_\infty R}{\nu}$
- $T$  Dimensional Temperature, [k].
- $u$  Dimensional axial velocity, [ m/s<sup>2</sup> ].
- $x,y$  Cartesian Coordinates, [m].

### Greek Symbols:

$\alpha$  Thermal diffusivity, [m<sup>2</sup>/s].

$\beta$  Volumetric coefficient of thermal expansion, [k<sup>-1</sup>].

$\xi$  Dimensionless surface location.

$\eta$  Dimensionless wall normal direction,  $\eta = \frac{y^*}{L}$

$\bar{\eta}$  modified normal direction,  $\bar{\eta} = \frac{y}{x} \left( \frac{Gr_x}{4} \right)^{1/4}$

$\chi$  wall normal direction at the leading-edge.

$\delta_{th}$  Thermal boundary layer thickness.

$\delta^*$  Modified thermal boundary layer thickness,

$$\delta^* = \frac{\delta_{th}}{\delta_{th}(flat\ plate)}.$$

$\nu$  Kinematic viscosity, [m<sup>2</sup>/s].

$\rho$  Density, [kg/m<sup>3</sup>].

$\phi$  Variable.

$\theta$  Dimensionless Temperature,  $\theta = \frac{T - T_\infty}{T_w - T_\infty}$ .

### Superscripts:

\* Dimensional quantity.

— Average quantity.

**Subscripts:**

$\gamma$  Under-relaxation coefficient.

# CONTENTS

DEDICATION.....	i
ACKNOWLEDGMENT .....	ii
TABLE OF CONTENTS .....	iii
LIST OF FIGURES .....	v
ABSTRACT .....	viii
NOMENCLATURE .....	ix
<b>CHAPTER ONE .....</b>	<b>1</b>
INTRODUCTION AND LITERATURE REVIEW.....	1
<b>CHAPTER TWO .....</b>	<b>8</b>
MATHEMATICAL FORMULATION .....	8
2.1 Introduction .....	8
2.2 Formulation of Governing Equations .....	9
2.3 Boundary Conditions .....	11
<b>CHAPTER THREE.....</b>	<b>13</b>
NUMERICAL METHOD OF SOLUTION .....	13
3.1 Introduction .....	13
3.2 Method of Solution .....	14
3.3 Steps of Solution.....	16
<b>CHAPTER FOUR .....</b>	<b>20</b>
RESULTS AND DISCUSSION .....	20
4.1 Introduction .....	20
4.2 Validation of the Results .....	21



4.3 Velocity Field.....	22
Effect of varying the nose radius of curvature ( $r$ ), and the fluid type ( $Pr$ ) on the axial velocity profile .....	22
4.4 Thermal Energy Field .....	24
4.3.1 Effect of varying the nose radius of curvature ( $r$ ), and the fluid type ( $Pr$ ) on the temperature profile.....	24
4.3.2 Effect of varying the nose radius of curvature, and the Prandtl number on the local and average Nusselt number distributions.....	27
<b>CHAPTER FIVE.....</b>	<b>31</b>
<b>CONCLUSIONS AND RECOMMENDATIONS.....</b>	<b>31</b>
5.1 Conclusions.....	31
5.2 Recommendations.....	32
<b>CHAPTER SIX.....</b>	<b>34</b>
<b>FIGURES</b>	
<b>BIBLIOGRAPHY.....</b>	<b>67</b>
<b>Arabic Abstract .....</b>	<b>69</b>

## LIST OF FIGURES

<u>Figures</u>	<u>Description</u>	<u>Page</u>
1	Schematic Diagram for Parabolic Body.....	35
2	Dimensionless Axial Velocity distribution , $r = 0.0$ (flat plate) $Pr = 0.71$ (Air).....	36
3	Dimensionless Temperature distribution, $r = 0.0$ (flat plate) $Pr = 0.71$ (Air).....	37
4	Pressure distribution on the surfaces of Parabolic bodies (Forced convection).....	38
5	Local skin Friction distribution on Parabolic bodies ( Forced Convection).....	39
6	Axial velocity distribution at $x = x_1, Pr = 0.71$ .....	40
7	Axial velocity distribution at $x = x_2, Pr = 0.71$ .....	41
8	Axial velocity distribution at $x = x_3, Pr = 0.71$ .....	42
9	Axial velocity distribution at $x = x_4, Pr = 0.71$ .....	43
10	Axial velocity distribution at $x = x_5, Pr = 0.71$ .....	44
11	Effect of wall temperature on local velocity Profile, $r = 0$ . (flat plate) $x = x_5, Pr = 0.71$ .....	45
12	Effect of wall temperature on local velocity Profile, $r = 0.001, x = x_5, Pr = 0.71$ .....	46
13	Effect of wall temperature on local velocity Profile, $r = 0.01, x = x_5, Pr = 0.71$ .....	47
14	Effect of wall temperature on local velocity Profile, $r = 0.1, x = x_5, Pr = 0.71$ .....	48
15	Effect of Prandtl number on local axial velocity distribution for a flat plate, $x = x_1$ .....	49

16	Effect of Prandtl number on the axial velocity distribution: parabola $r = 0.001$ , $x=x_1$ .....	50
17	Effect of Prandtl number on the axial velocity distribution: parabola $r = 0.01$ , $x=x_1$ .....	51
18	Effect of Prandtl number on the axial velocity distribution: a parabola $r = 0.1$ , $x=x_1$ .....	52
19	Effect of nose radius of curvature on local temperature distribution: $x = x_1$ , $Pr = 0.71$ (Air).....	53
20	Effect of nose radius of curvature on local temperature distribution: $x = x_2$ , $Pr = 0.71$ (Air).....	54
21	Effect of nose radius of curvature on local temperature distribution : $x = x_3$ , $Pr = 0.71$ (Air).....	55
22	Effect of nose radius of curvature on local temperature distribution: $x = x_4$ , $Pr = 0.71$ (Air).....	56
23	Effect of nose radius of curvature on local temperature distribution: $x = x_5$ , $Pr = 0.71$ (Air).....	57
24	Effect of Prandtl number (Pr) on the local temperature distribution: $r = 0$ . (flat plate), $x = x_1$ .....	58
25	Effect of Prandtl number (Pr) on the local temperature distribution: $r = 0.001$ , $x = x_1$ .....	59
26	Effect of Prandtl number ( Pr ) on the local temperature distribution: $r = 0.01$ , $x = x_1$ .....	60
27	Effect of Prandtl number ( Pr ) on the local temperature distribution: $r = 0.1$ , $x = x_1$ .....	61
28	Temperature distribution at the lading-edge of the parabola, $r =$ $0.001$ , $Pr =0.71$ .....	62
29	Thermal boundary layer thickness for different Parabola shapes.....	63

30	Modified thermal boundary layer thickness for different parabola shapes, $Pr = 0.71$ .....	64
31	Local Nusselt number distribution for different parabolic surfaces.....	65
32	Average Nusselt number distribution for different parabola Shapes.....	66

Natural Convection Flow Over A Parabolic Cylinder	العنوان:
Bany Youness, Asad Aldeen M. H.	المؤلف الرئيسي:
Haddad, Osamah Menwer(super)	مؤلفين آخرين:
2000	التاريخ الميلادي:
عمان، الأردن	موقع:
1 - 69	الصفحات:
568665	رقم MD:
رسائل جامعية	نوع المحتوى:
Arabic	اللغة:
رسالة ماجستير	الدرجة العلمية:
جامعة العلوم والتكنولوجيا الاردنية	الجامعة:
كلية الدراسات العليا	الكلية:
الاردن	الدولة:
Dissertations	قواعد المعلومات:
اسطوانات الحرارة، الفيزياء، الهندسة الميكانيكية	مواضيع:
<a href="https://search.mandumah.com/Record/568665">https://search.mandumah.com/Record/568665</a>	رابط:

***Natural Convection Flow over a Parabolic  
Cylinder***

***at***

**Jordan University of Science and Technology.**

May, 2000

# *Natural Convection Flow Over a Parabolic Cylinder*

By

Asad Aldeen M. H. Bany-Youness

Thesis submitted in partial fulfillment of the requirements for the degree  
of M.Sc. in mechanical engineering

at

Faculty of Graduate Studies

Jordan University of Science and Technology

May, 2000

Signature of Author *May 28, 2000* *Asad*

Committee Members

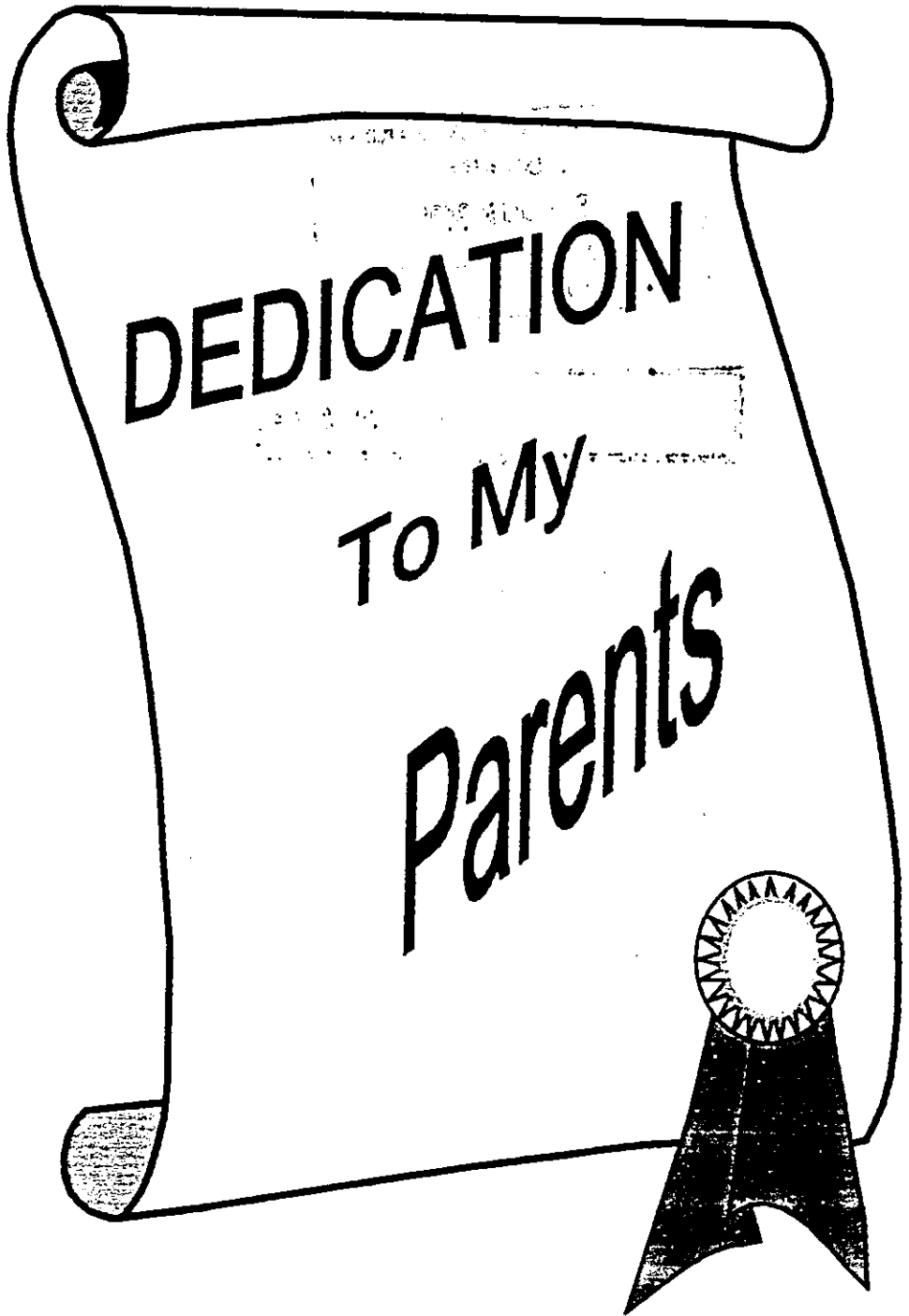
Date And Signature

Dr. Osamah Haddad, Chairman... *May 30, 2000*..... *O.M. HADDAD*

Prof. Taha K. Aldoss..... *31/5/2000*.....

Dr. Mohammad Al-Nimr..... *5/31/2000*..... *Moh'd Al. Nimr*

Dr. Jamal M. Nazzal (Cognate, Amman University)..... *Jamall*..... *5/6/2000*





## **ACKNOWLEDGMENT**

I would like to extend my sincere regards and gratitude to my supervisor Dr. Osamah Haddad for his invaluable assistance, guidance, and encouragement. Special thanks are also expressed to the committee members Professor Taha Aldoss, Dr. Mohammad Al-Nimr, and Dr. Jamal M. Nazzal.

I shall always be grateful for my colleagues for their endless encouragement and for their continuous support.

Finally, I would like to express my deep gratitude and great respect to my parents for their long patience with my endless demands and for their valuable support.

# CONTENTS

DEDICATION.....	i
ACKNOWLEDGMENT .....	ii
TABLE OF CONTENTS .....	iii
LIST OF FIGURES .....	v
ABSTRACT .....	viii
NOMENCLATURE .....	ix
<b>CHAPTER ONE .....</b>	<b>1</b>
INTRODUCTION AND LITERATURE REVIEW.....	1
<b>CHAPTER TWO .....</b>	<b>8</b>
MATHEMATICAL FORMULATION .....	8
2.1 Introduction .....	8
2.2 Formulation of Governing Equations .....	9
2.3 Boundary Conditions .....	11
<b>CHAPTER THREE.....</b>	<b>13</b>
NUMERICAL METHOD OF SOLUTION .....	13
3.1 Introduction .....	13
3.2 Method of Solution .....	14
3.3 Steps of Solution.....	16
<b>CHAPTER FOUR .....</b>	<b>20</b>
RESULTS AND DISCUSSION .....	20
4.1 Introduction .....	20
4.2 Validation of the Results .....	21

4.3 Velocity Field.....	22
Effect of varying the nose radius of curvature ( $r$ ), and the fluid type ( $Pr$ ) on the axial velocity profile .....	22
4.4 Thermal Energy Field .....	24
4.3.1 Effect of varying the nose radius of curvature ( $r$ ), and the fluid type ( $Pr$ ) on the temperature profile.....	24
4.3.2 Effect of varying the nose radius of curvature, and the Prandtl number on the local and average Nusselt number distributions.....	27
<b>CHAPTER FIVE.....</b>	<b>31</b>
<b>CONCLUSIONS AND RECOMMENDATIONS.....</b>	<b>31</b>
5.1 Conclusions.....	31
5.2 Recommendations.....	32
<b>CHAPTER SIX.....</b>	<b>34</b>
<b>FIGURES</b>	
<b>BIBLIOGRAPHY.....</b>	<b>67</b>
<b>Arabic Abstract .....</b>	<b>69</b>

## LIST OF FIGURES

<u>Figures</u>	<u>Description</u>	<u>Page</u>
1	Schematic Diagram for Parabolic Body.....	35
2	Dimensionless Axial Velocity distribution , $r = 0.0$ (flat plate) $Pr = 0.71$ (Air).....	36
3	Dimensionless Temperature distribution, $r = 0.0$ (flat plate) $Pr = 0.71$ (Air).....	37
4	Pressure distribution on the surfaces of Parabolic bodies (Forced convection).....	38
5	Local skin Friction distribution on Parabolic bodies ( Forced Convection).....	39
6	Axial velocity distribution at $x = x_1, Pr = 0.71$ .....	40
7	Axial velocity distribution at $x = x_2, Pr = 0.71$ .....	41
8	Axial velocity distribution at $x = x_3, Pr = 0.71$ .....	42
9	Axial velocity distribution at $x = x_4, Pr = 0.71$ .....	43
10	Axial velocity distribution at $x = x_5, Pr = 0.71$ .....	44
11	Effect of wall temperature on local velocity Profile, $r = 0$ . (flat plate) $x = x_5, Pr = 0.71$ .....	45
12	Effect of wall temperature on local velocity Profile, $r = 0.001, x = x_5, Pr = 0.71$ .....	46
13	Effect of wall temperature on local velocity Profile, $r = 0.01, x = x_5, Pr = 0.71$ .....	47
14	Effect of wall temperature on local velocity Profile, $r = 0.1, x = x_5, Pr = 0.71$ .....	48
15	Effect of Prandtl number on local axial velocity distribution for a flat plate, $x = x_1$ .....	49

16	Effect of Prandtl number on the axial velocity distribution: parabola $r = 0.001$ , $x=x_1$ .....	50
17	Effect of Prandtl number on the axial velocity distribution: parabola $r = 0.01$ , $x=x_1$ .....	51
18	Effect of Prandtl number on the axial velocity distribution: a parabola $r = 0.1$ , $x=x_1$ .....	52
19	Effect of nose radius of curvature on local temperature distribution: $x = x_1$ , $Pr = 0.71$ (Air).....	53
20	Effect of nose radius of curvature on local temperature distribution: $x = x_2$ , $Pr = 0.71$ (Air).....	54
21	Effect of nose radius of curvature on local temperature distribution : $x = x_3$ , $Pr = 0.71$ (Air).....	55
22	Effect of nose radius of curvature on local temperature distribution: $x = x_4$ , $Pr = 0.71$ (Air).....	56
23	Effect of nose radius of curvature on local temperature distribution: $x = x_5$ , $Pr = 0.71$ (Air).....	57
24	Effect of Prandtl number ( $Pr$ ) on the local temperature distribution: $r = 0$ . (flat plate), $x = x_1$ .....	58
25	Effect of Prandtl number ( $Pr$ ) on the local temperature distribution: $r = 0.001$ , $x = x_1$ .....	59
26	Effect of Prandtl number ( $Pr$ ) on the local temperature distribution: $r = 0.01$ , $x = x_1$ .....	60
27	Effect of Prandtl number ( $Pr$ ) on the local temperature distribution: $r = 0.1$ , $x = x_1$ .....	61
28	Temperature distribution at the lading-edge of the parabola, $r =$ $0.001$ , $Pr = 0.71$ .....	62
29	Thermal boundary layer thickness for different Parabola shapes.....	63

30	Modified thermal boundary layer thickness for different parabola shapes, $Pr = 0.71$ .....	64
31	Local Nusselt number distribution for different parabolic surfaces.....	65
32	Average Nusselt number distribution for different parabola Shapes.....	66

# Natural Convection Flow Over a Parabolic Cylinder

By: Asad Al-deen M. H. Bany-Youness

Advisor : Dr. Osamah Haddad

## ABSTRACT

Steady, two-dimensional, symmetric and incompressible Natural convection flow over a parabolic cylinder was investigated numerically. The full Navier-Stokes and the energy equations were considered. These equations has been solved using a commercial code called 'COSMOS' which is based on finite element. Solutions for the temperature and velocity distributions are obtained for different values of the flow parameters. In addition, the local and average Nusselt number distributions are obtained and presented. For all cases, the surface was considered to be isothermal, and the following parameters are studied: nose radius of curvature of the parabolic body, Grashof number, and Prandtl number.

It was found that as the nose radius of curvature of the parabolic body is increased, the flow velocity will enhanced, but the local and average Nusselt number is decreased. Also, the increase in Grashof or Prandtl numbers will increase the local and average Nusselt number.

## NOMENCLATURE

- $A_{ii}$  The algebraic coefficients.
- $C_f$  Local Skin friction,  $C_f = \frac{\tau_w}{\rho U_\infty^2}$ .
- $g$  Gravitational acceleration, [m/s<sup>2</sup>].
- $Gr_x$  Local Grashof number,  $Gr_x = \frac{g\beta \Delta T x^3}{\nu^2}$ .
- $h$  Local heat transfer coefficient, [w/ m k].
- $L$  Characteristic length of the parabola.
- $Nu$  Local Nusselt number at the surface of the parabola.
- $\overline{Nu}$  Average Nusselt number,  $\overline{Nu} = \frac{1}{L} \int_0^L Nu_x dx$ .
- $Pr$  Prandtl number,  $Pr = \frac{\nu}{\alpha}$ .
- $R$  Nose radius of curvature.
- $Re$  Reynolds number based on nose radius of curvature,  $Re = \frac{U_\infty R}{\nu}$
- $T$  Dimensional Temperature, [k].
- $u$  Dimensional axial velocity, [ m/s<sup>2</sup> ].
- $x,y$  Cartesian Coordinates, [m].



## Greek Symbols:

$\alpha$  Thermal diffusivity, [m<sup>2</sup>/s].

$\beta$  Volumetric coefficient of thermal expansion, [k<sup>-1</sup>].

$\xi$  Dimensionless surface location.

$\eta$  Dimensionless wall normal direction,  $\eta = \frac{y^*}{L}$

$\bar{\eta}$  modified normal direction,  $\bar{\eta} = \frac{y}{x} \left( \frac{Gr_x}{4} \right)^{1/4}$

$\chi$  wall normal direction at the leading-edge.

$\delta_{th}$  Thermal boundary layer thickness.

$\delta^*$  Modified thermal boundary layer thickness,

$$\delta^* = \frac{\delta_{th}}{\delta_{th}(flat\ plate)}.$$

$\nu$  Kinematic viscosity, [m<sup>2</sup>/s].

$\rho$  Density, [kg/m<sup>3</sup>].

$\phi$  Variable.

$\theta$  Dimensionless Temperature,  $\theta = \frac{T - T_\infty}{T_w - T_\infty}$ .

## Superscripts:

\* Dimensional quantity.

— Average quantity.

**Subscripts:**

$\gamma$  Under-relaxation coefficient.

# CHAPTER ONE

## INTRODUCTION AND LITERATURE REVIEW

Most of the fluid motions and transport that affect our lives, our immediate surroundings, and our near and far environment are induced by buoyancy. Such flows are found in the air circulation around our bodies, in the enclosures we frequent, in cooking, in processing, in pools of water, and in atmospheric, lake, and oceanic circulation at every scale. They are found inside planetary bodies and are presumed to occur in and around celestial ones. The buoyancy force arises from motive density differences resulting from inhomogeneities in temperature, differences in concentrations of chemical species, changes in material phase, and many other effects. Different kinds of buoyancy-induced flows are found because of the various separate effects and their combinations, the occurrence of many different geometric configurations, and the different bounding conditions and force fields that arise.

Buoyancy induced motions are initially laminar at a small scale. However, the vigorous flows at larger scales are inevitably turbulent. Laminar and turbulent transport characteristics usually differ widely in

buoyancy-induced flows, as in the forced ones. It is interesting that the flows in water and in atmospheric air, at the human size scale, are initially laminar, before perhaps beginning a transition toward downstream turbulence. Laminar transport characteristically persists to a distance of order of 1 m, more or less, depending on bounding conditions. Therefore, the many flows of smaller scale are treated as laminar. For those at a much larger scale, the initial laminar portion may often be ignored and the whole flow field treated as turbulent. At scales between these there are important questions of when and how becomes unstable and eventually proceeds to turbulence. In recent years there has been a rapid increase in the intensity of research in this field. Some of this effort represents a shift in interest from several conventional fields of fluid mechanics of diminished relative importance. However, most of it is due to a growing demand for detailed quantitative information concerning buoyancy-induced motions in the atmosphere, bodies of water, quasi-solid bodies such as the earth, enclosures, and devices of process equipment. The result has been a rapid expansion of knowledge in areas little considered only a few years ago.

The flow around a parabolic body is of a great importance in practice which have received little attention [1]. For example, in

Aerospace the aerodynamic bodies designed for subsonic flow generally have finite thickness distributions with a parabolic leading edge [2]. Also, in Turbomachinery applications, the cross-section of blades is usually identical with that of an airfoil. The objective of this study is to solve the Navier-Stokes and energy equations which govern the natural convection flow from a parabola. An attempt will be made to cover a wide range of flow parameters (e.g. nose radius of curvature, Prandtl number (Pr)). Solution will be checked against the flat plate solution, which can be obtained when the nose radius of curvature of the parabola is equal zero.

All past solutions of the flow over a parabola have considered only the hydrodynamic part of the problem [1,3,4,5,6 and 7]. The only exception to this is the numerical study carried out by Haddad et al. [8] in which the energy equation was solved with the Navier-Stokes equations for forced convection laminar flow from parabolic bodies. The goal of this study is to solve the Navier-Stokes and energy equations that govern the natural convection heat transfer from parabolic cylinders. The effect of the different flow parameters (e.g. Grashof number, Prandtl number, nose radius of curvature ) will be investigated. Finally, Nusselt number distributions will be obtained and all results

relevant to the parabolic body will be compared with the flat plate results.

Davis [1] used parabolic coordinates to solve the laminar incompressible flow past a parabolic cylinder using different values of Reynolds number. In the limit as the Reynolds number based on the nose radius of curvature goes to zero, the solution for the flow past a semi-infinite flat plate is obtained. All solutions are found by using an implicit alternating direction method to solve the time-dependent Navier-Stokes equations. Davis focused careful attention on extracting the singularities from the problem in the limit as Reynolds number goes to zero ( i.e. flat plate solution ). The same flow problem was treated also by Dennis and Walsh [4] using finite difference approximations to the partial differential equations for the stream function and vorticity as dependent variables. In their study, Dennis and Walsh were not able to get a solution for Reynolds number smaller than 0.25 because of singularity problem, although their results were in good agreement with those of Davis [1]. They had a small but significant difference between their results and those of the second order boundary layer approximation especially in skin friction.

The flow past a semi-infinite flat plate using truncation series or local similarity method applied on full Navier-Stokes equations was investigated by Davis [5]. The difficulty of matching approximations of low Reynolds number (Stokes) approximation with high Reynolds number (Boundary Layer) approximation is avoided, and solutions are obtained for all values of Reynolds number. Van De Vooren and Dijkstra [6] applied the same approach used by Davis [5] to solve numerically the nature of the flow near the leading edge of flat plate. They used simple finite difference expressions and the system of equations was solved by iterative technique. Their solution was also valid for any value of Reynolds number. They have shown that there is about 5% error near the leading edge in skin friction in the results of Davis [5].

The method of Van De Vooren and Dijkstra [6] for semi-infinite flat plate was extended to the case of parabolic cylinder by Botta, Dijkstra and Veldman [7]. They managed to extend the solution for the case of Reynolds number approaches infinity (boundary layer). The drag coefficient have been checked by means of application of the momentum theory to an infinitely large circular contour and the deviation was within 2% for smallest mesh size.

Numerical Solutions of the Navier-Stokes and Energy equations for Laminar Incompressible Flow Past Parabolic Bodies is investigated by Haddad, et.al [8]. The full Navier-Stokes and energy equations in parabolic coordinates with stream function, vorticity and temperature as dependent variables were solved. These equations were linearized using Newton's linearization technique. The resulting set of equations were solved using a second order accurate finite difference scheme on a non-uniform grid. Results were presented for pressure, velocity and temperature distributions in addition to local and average skin friction distributions. The effect of both Reynolds number and Prandtl number on the local and average Nusselt number is also presented. The obtained solutions were compared with the previous solutions in the literature showing good agreement with them.

Kuehn and Goldstein [9] studied the laminar natural-convection heat transfer from horizontal isothermal circular cylinder by solving the Navier-Stokes and energy equations using an elliptic numerical procedure. Results are obtained for  $10^0 \leq Ra \leq 10^7$ . The flow approaches natural convection from a line heat source as  $Ra \rightarrow 0$  and laminar boundary layer flow as  $Ra \rightarrow \infty$ . Boundary layer solutions do not adequately describes the flow and heat transfer at low or moderate values



of Ra because of the neglect of curvature effects and the breakdown of the boundary layer assumptions in the region of the plume. Kuehn and Goldstein achieved good agreement with experimental results.

Finally, as it can be seen in the pre-mentioned literature there exist no experiment relevant to natural convection over a parabolic cylinder, nor a theoretical or numerical study, which considers this type of flow.

# CHAPTER TWO

## MATHEMATICAL FORMULATION

### 2.1 Introduction

A schematic diagram of the problem under consideration is shown in Figure 1. The equation of the surface of the parabolic body is given by

$$x(y) = \frac{1}{2R}(y^2 - R^2) \quad (2.1)$$

based on this, the radius of curvature  $r(x)$  of the parabola  $x(y)$  is given by :

$$r(x) = \frac{(1 + x'^2)^{3/2}}{x''} = \left[ \frac{8(R+x)^3}{R} \right]^{1/2} \quad (2.2)$$

at the nose where  $y = 0$  and  $x = -R/2$  one can get from equation (2.1) that the radius of curvature is

$$r(x = -R/2) = R \quad (2.3)$$

thus,  $R$  is recognized as the nose radius of curvature.

## 2.2 Formulation of Governing Equations

The full Navier – Stokes ( N-S ) and energy equations for 2-D laminar steady incompressible with constant viscosity fluid flow in the Cartesian coordinates system take the form:

Continuity equation:

$$\frac{\partial \rho u^*}{\partial x^*} + \frac{\partial \rho v^*}{\partial y^*} = 0. \quad (2.4)$$

X\_momentum equation:

$$\rho u^* \frac{\partial u^*}{\partial x^*} + \rho v^* \frac{\partial u^*}{\partial y^*} = \rho g - \frac{\partial P^*}{\partial x^*} + \mu \left[ \frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right] \quad (2.5)$$

Y\_momentum equation:

$$\rho u^* \frac{\partial v^*}{\partial x^*} + \rho v^* \frac{\partial v^*}{\partial y^*} = -\frac{\partial P^*}{\partial y^*} + \mu \left[ \frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right] \quad (2.6)$$

Energy equation:

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \alpha \left( \frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} \right) \quad (2.7)$$

it is assumed that there is no heat source or sink, the thermophysical properties are constant, and the viscous dissipation is negligible. In order

to generalize the results, the following non-dimensional variables are introduced:

$$x = \frac{x^*}{l}, y = \frac{y^*}{l}, v = \frac{v^*}{\nu/l}, u = \frac{u^*}{\nu/l}$$

$$\theta = \frac{T^* - T_{\infty}^*}{T_w^* - T_{\infty}^*}, P = \frac{P^* - P_{\infty}^*}{\rho \nu^2 / l^2} \quad (2.8)$$

substituting these variables back into equations (2.4,2.5,2.6 and 2.7), using the Boussinesq approximation leads to the following dimensionless set of equations:

Continuity equation :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.0 \quad (2.9)$$

X\_momentum equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = Gr^* \theta + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \quad (2.10)$$

Y\_momentum equation:

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \quad (2.11)$$

Energy equation :

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \quad (2.12)$$

where (Gr) is the Grashof number and (Pr) is the Prantdl number. Equations (2.9,2.10,2.11,and 2.12) form a system of non-linear, elliptic partial differential equations.

### **2.3 Boundary Conditions:**

For an elliptic system of equations, boundary conditions for the unknown variables must be specified on all boundaries of the flow domain. Since the x-axis is an axis of symmetry (Figure 1), only one half of the domain can be considered as the solution domain. Also the required boundary condition is been specified. At the wall, the no-slip no-penetration ( $u =0, v =0$ ) are applied at an isothermal wall. Far down stream, the temperature is ambient, and the axial velocity is set to be zero. These boundary conditions are sufficient to achieve a convergent solution.

The above boundary can be summarized as follows :

#### **- Wall boundary conditions:**

$$\text{- at } y=y_w \quad u=0, v=0 \text{ and } \theta = 1.0 \quad (2.13)$$

#### **- Free stream boundary conditions :**

$$\text{- as } y \rightarrow \infty \quad u = 0.0 \text{ and } \theta = 0.0 \quad (2.14)$$

Natural Convection Flow Over A Parabolic Cylinder	العنوان:
Bany Youness, Asad Aldeen M. H.	المؤلف الرئيسي:
Haddad, Osamah Menwer(super)	مؤلفين آخرين:
2000	التاريخ الميلادي:
عمان، الأردن	موقع:
1 - 69	الصفحات:
568665	رقم MD:
رسائل جامعية	نوع المحتوى:
Arabic	اللغة:
رسالة ماجستير	الدرجة العلمية:
جامعة العلوم والتكنولوجيا الاردنية	الجامعة:
كلية الدراسات العليا	الكلية:
الاردن	الدولة:
Dissertations	قواعد المعلومات:
اسطوانات الحرارة، الفيزياء، الهندسة الميكانيكية	مواضيع:
<a href="https://search.mandumah.com/Record/568665">https://search.mandumah.com/Record/568665</a>	رابط:

***Natural Convection Flow over a Parabolic  
Cylinder***

***at***

**Jordan University of Science and Technology.**

May, 2000

# *Natural Convection Flow Over a Parabolic Cylinder*

By

Asad Aldeen M. H. Bany-Youness

Thesis submitted in partial fulfillment of the requirements for the degree  
of M.Sc. in mechanical engineering

at

Faculty of Graduate Studies

Jordan University of Science and Technology

May, 2000

Signature of Author *May 28, 2000* *Asad*

Committee Members

Date And Signature

Dr. Osamah Haddad, Chairman... *May 30, 2000*..... O.M. HADDAD

Prof. Taha K. Aldoss..... *31/5/2000*.....

Dr. Mohammad Al-Nimr..... *5/31/2000*..... *Moh'd Al. Nimr*

Dr. Jamal M. Nazzal (Cognate, Amman University)..... *Jamall*..... *5/6/2000*